1-1 Expressions and Formulas

Main Ideas

- Use the order of operations to evaluate expressions.
- Use formulas.

New Vocabulary

variable
algebraic expression
order of operations
monomial
constant
coefficient
degree
power
polynomial
term
like terms
trinomial
binomial
formula

GET READY for the Lesson

Nurses setting up intravenous or IV fluids must control the flow rate F, in drops per minute. They use the formula $F = \frac{V \times d}{t}$, where V is the volume of the solution in milliliters, d is the drop factor in drops per milliliter, and t is the time in minutes.

Suppose 1500 milliliters of saline are to be given over 12 hours. Using a drop factor of 15 drops per milliliter, the expression $\frac{1500 \times 15}{12 \times 60}$ gives the correct IV flow rate.

Order of Operations

Variables are symbols, usually letters, used to represent unknown quantities. Expressions that contain at least one variable are called **algebraic expressions**. You can evaluate an algebraic expression by replacing each variable with a number and then applying the **order of operations**.

KEY CONCEPT *Order of Operations*

Step 1 Evaluate expressions inside grouping symbols.
Step 2 Evaluate all powers.
Step 3 Multiply and/or divide from left to right.
Step 4 Add and/or subtract from left to right.

An algebraic expression that is a number, a variable, or the product of a number and one or more variables is called a **monomial**. Monomials cannot contain variables in denominators, variables with exponents that are negative, or variables under radicals.

Monomials	Not Monomials
$5b$	$\frac{1}{n^4}$
$-w$	$\sqrt[3]{x}$
23	$x + 8$
x^2	a^{-1}
$\frac{1}{3}x^3y^4$	

Constants are monomials that contain no variables, like 23 or -1. The numerical factor of a monomial is the **coefficient** of the variable(s). For example, the coefficient of m in $-6m$ is -6. The **degree** of a monomial is the sum of the exponents of its variables. For example, the degree of $12g^7h^4$ is $7 + 4$ or 11. The degree of a constant is 0. A **power** is an expression of the form x^n. The word *power* is also used to refer to the exponent itself.

A **polynomial** is a monomial or a sum of monomials. The monomials that make up a polynomial are called the **terms** of the polynomial. In a polynomial such as $x^2 + 2x + x + 1$, the two monomials $2x$ and x can be combined because they are **like terms**. The result is $x^2 + 3x + 1$. The polynomial $x^2 + 3x + 1$ is a **trinomial** because it has three unlike terms. A polynomial such as $xy + z^3$ is a **binomial** because it has two unlike terms.

EXAMPLE Evaluate Algebraic Expressions

1 **a. Evaluate $m + (n - 1)^2$ if $m = 3$ and $n = -4$.**

$$\begin{aligned} m + (n - 1)^2 &= 3 + (-4 - 1)^2 && \text{Replace } m \text{ with 3 and } n \text{ with } -4. \\ &= 3 + (-5)^2 && \text{Add } -4 \text{ and } -1. \\ &= 3 + 25 && \text{Find } (-5)^2. \\ &= 28 && \text{Add 3 and 25.} \end{aligned}$$

b. Evaluate $x^2 - y(x + y)$ if $x = 8$ and $y = 1.5$.

$$\begin{aligned} x^2 - y(x + y) &= 8^2 - 1.5(8 + 1.5) && \text{Replace } x \text{ with 8 and } y \text{ with 1.5.} \\ &= 8^2 - 1.5(9.5) && \text{Add 8 and 1.5.} \\ &= 64 - 1.5(9.5) && \text{Find } 8^2. \\ &= 64 - 14.25 && \text{Multiply 1.5 and 9.5.} \\ &= 49.75 && \text{Subtract 14.25 from 64.} \end{aligned}$$

c. Evaluate $\dfrac{a^3 + 2bc}{c^2 - 5}$ if $a = 2$, $b = -4$, and $c = -3$.

$$\begin{aligned} \frac{a^3 + 2bc}{c^2 - 5} &= \frac{2^3 + 2(-4)(-3)}{(-3)^2 - 5} && a = 2, b = -4, \text{ and } c = -3 \\ &= \frac{8 + (-8)(-3)}{9 - 5} && \text{Evaluate the numerator and the denominator separately.} \\ &= \frac{8 + 24}{9 - 5} && \text{Multiply } -8 \text{ by } -3. \\ &= \frac{32}{4} \text{ or } 8 && \text{Simplify the numerator and the denominator. Then divide.} \end{aligned}$$

Study Tip

Fraction Bar

The fraction bar acts as both an operation symbol, indicating division, and as a grouping symbol. Evaluate the expressions in the numerator and denominator separately before dividing.

CHECK Your Progress

1A. Evaluate $m + (3 - n)^2$ if $m = 12$ and $n = -1$.

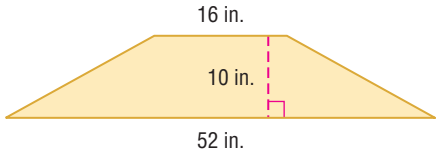
1B. Evaluate $x^2y + x(x - y)$ if $x = 4$ and $y = 0.5$.

1C. Evaluate $\dfrac{b^2 - 3a^2c}{b^3 + 2}$ if $a = -1$, $b = 2$, and $c = 8$.

Formulas A **formula** is a mathematical sentence that expresses the relationship between certain quantities. If you know the value of every variable in the formula except one, you can find the value of the remaining variable.

EXAMPLE 2 Use a Formula

GEOMETRY **The formula for the area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h represents the height, and b_1 and b_2 represent the measures of the bases. Find the area of the trapezoid shown below.**



The height is 10 inches. The bases are 16 inches and 52 inches. Substitute each value given into the formula. Then evaluate the expression using the order of operations.

$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$= \frac{1}{2}(10)(16 + 52)$ Replace h with 10, b_1 with 16, and b_2 with 52.

$= \frac{1}{2}(10)(68)$ Add 16 and 52.

$= 5(68)$ Multiply $\frac{1}{2}$ and 10.

$= 340$ Multiply 5 by 68.

The area of the trapezoid is 340 square inches.

CHECK Your Progress

2. The formula for the volume V of a rectangular prism is $V = \ell wh$, where ℓ represents the length, w represents the width, and h represents the height. Find the volume of a rectangular prism with a length of 4 feet, a width of 2 feet, and a height of 3.5 feet.

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CHECK Your Understanding

Example 1 (p. 7)

Evaluate each expression if $x = 4$, $y = -2$, and $z = 3.5$.

1. $z - x + y$

2. $x + (y - 1)^3$

3. $x + [3(y + z) - y]$

4. $\frac{x^2 - y}{z + 2.5}$

5. $\frac{x + 2y^2}{x - z}$

6. $\frac{y^3 + 2xz}{x^2 - z}$

Example 2 (p. 8)

BANKING **For Exercises 7 and 8, use the following information.**
Simple interest is calculated using the formula $I = prt$, where p represents the principal in dollars, r represents the annual interest rate, and t represents the time in years. Find the simple interest I given each set of values.

7. $p = \$1800$, $r = 6\%$, $t = 4$ years

8. $p = \$31{,}000$, $r = 2\frac{1}{2}\%$, $t = 18$ months

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–22	1
23, 24	2

Evaluate each expression if $w = 6$, $x = 0.4$, $y = \frac{1}{2}$, and $z = -3$.

9. $w + x + z$ **10.** $w + 12 \div z$ **11.** $w(8 - y)$

12. $z(x + 1)$ **13.** $w - 3x + y$ **14.** $5x + 2z$

Evaluate each expression if $a = 3$, $b = 0.3$, $c = \frac{1}{3}$, and $d = -1$.

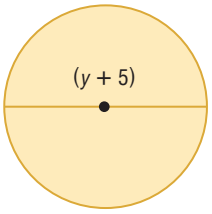
15. $\frac{a - d}{bc}$ **16.** $\frac{a + d}{c}$ **17.** $\frac{a^2c^2}{d}$

18. $\frac{a - 10b}{c^2d^2}$ **19.** $\frac{d + 4}{a^2 + 3}$ **20.** $\frac{1 - b}{3c - 3b}$

21. NURSING Determine the IV flow rate for the patient described at the beginning of the lesson by finding the value of $\frac{1500 \times 15}{12 \times 60}$.

22. BICYCLING Air pollution can be reduced by riding a bicycle rather than driving a car. To find the number of pounds of pollutants created by starting a typical car 10 times and driving it for 50 miles, find the value of the expression $\frac{(52.84 \times 10) + (5.955 \times 50)}{454}$.

23. GEOMETRY The formula for the area A of a circle with diameter d is $A = \pi\left(\frac{d}{2}\right)^2$. Write an expression to represent the area of the circle.



24. GEOMETRY The formula for the volume V of a right circular cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$. Write an expression for the volume of a cone with $r = 3x$ and $h = 2x$.

Evaluate each expression if $a = \frac{2}{5}$, $b = -3$, $c = 0.5$, and $d = 6$.

25. $b^4 - d$ **26.** $(5 - d)^2 + a$ **27.** $\frac{5ad}{b}$

28. $\frac{2b - 15a}{3c}$ **29.** $(a - c)^2 - 2bd$ **30.** $\frac{1}{c} + \frac{1}{d}$

31. Find the value of ab^n if $n = 3$, $a = 2000$, and $b = -\frac{1}{5}$.

Real-World Link

To estimate the width w in feet of a firework burst, use the formula $w = 20At$. In this formula, A is the estimated viewing angle of the fireworks display, and t is the time in seconds from the instant you see the light until you hear the sound.

Source: efg2.com

32. FIREWORKS Suppose you are about a mile from a fireworks display. You count 5 seconds between seeing the light and hearing the sound of the fireworks display. You estimate the viewing angle is about 4°. Using the information at the left, estimate the width of the firework display.

33. MONEY In 1960, the average price of a car was about $2500. This may sound inexpensive, but the average income in 1960 was much less than it is now. To compare dollar amounts over time, use the formula $V = \frac{A}{S}C$, where A is the old dollar amount, S is the starting year's Consumer Price Index (CPI), C is the converting year's CPI, and V is the current value of the old dollar amount. Buying a car for $2500 in 1960 was like buying a car for how much money in 2004?

Year	1960	1970	1980	1990	2000	2004
Average CPI	29.6	38.8	82.4	130.7	172.2	188.9

Source: U.S. Department of Labor

EXTRA PRACTICE
See pages 891, 926.
Math Online
Self-Check Quiz at algebra2.com

34. MEDICINE A patient must take blood pressure medication that is dispensed in 125-milligram tablets. The dosage is 15 milligrams per kilogram of body weight and is given every 8 hours. If the patient weighs 25 kilograms, how many tablets would be needed for a 30-day supply? Use the formula $n = [15b \div (125 \times 8)] \times 24d$, where n is the number of tablets, d is the number of days the supply should last, and b is body weight in kilograms.

35. QB RATING The formula for quarterback efficiency rating in the National Football League is $\left(\frac{\frac{C}{A} - 0.3}{0.2} + \frac{\frac{Y}{A} - 3}{4} + \frac{\frac{T}{A}}{0.05} + \frac{0.095 - \frac{I}{A}}{0.04}\right) \times \frac{100}{6}$, where C is the number of passes completed, A is the number of passes attempted, Y is passing yardage, T is the number of touchdown passes, and I is the number of interceptions. In 2005, Ben Roethlisberger of the Pittsburgh Steelers completed 168 of the 268 passes he attempted for 2385 yards. He threw 17 touchdowns and 9 interceptions. Find his efficiency rating for 2005.

H.O.T. Problems

36. OPEN ENDED Write an algebraic expression in which subtraction is performed before division, and the symbols ( ), [ ], or { } are not used.

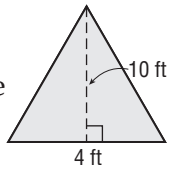
37. CHALLENGE Write expressions having values from one to ten using exactly four 4s. You may use any combination of the operation symbols +, −, ×, ÷, and/or grouping symbols, but no other digits are allowed. An example of such an expression with a value of zero is $(4 + 4) - (4 + 4)$.

38. REASONING Explain how to evaluate $a + b[(c + d) \div e]$, if you were given the values for a, b, c, d, and e.

39. *Writing in Math* Use the information about IV flow rates on page 6 to explain how formulas are used by nurses. Explain why a formula for the flow rate of an IV is more useful than a table of specific IV flow rates and describe the impact of using a formula, such as the one for IV flow rate, incorrectly.

STANDARDIZED TEST PRACTICE

40. ACT/SAT The following are the dimensions of four rectangles. Which rectangle has the same area as the triangle at the right?



A 1.6 ft by 25 ft
B 5 ft by 16 ft
C 3.5 ft by 4 ft
D 0.4 ft by 50 ft

41. REVIEW How many cubes that are 3 inches on each edge can be placed completely inside a box that is 9 inches long, 6 inches wide, and 27 inches tall?

F 12
G 36
H 54
J 72

GET READY for the Next Lesson

PREREQUISITE SKILL Evaluate each expression.

42. $\sqrt{9}$
43. $\sqrt{16}$
44. $\sqrt{100}$
45. $\sqrt{169}$
46. $-\sqrt{4}$
47. $-\sqrt{25}$
48. $\sqrt{\frac{4}{9}}$
49. $\sqrt{\frac{36}{49}}$